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Short-Term Resource Scheduling with Ramp Constraints

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Abstract—This paper describes a Lagrangian relaxation based method to solve the short-term resource scheduling (STRS) problem with ramp constraints. Instead of discretizing the generation levels, the ramp rate constraints are relaxed with the system demand constraints using Lagrange multipliers. Three kinds of ramp constraints, startup, operating and shutdown ramp constraints are considered. The proposed method has been applied to solve the hydro-thermal generation scheduling problem at PG&E. An example along with numerical results is also presented.

I Introduction

In a multi-area hydrothermal power system like PG&E's, the short-term resource scheduling (STRS) problem is solved on a daily basis to determine major unit commitment and transaction decisions, set the parameters of hydro scheduling, and evaluate participation in spot energy markets. Ramping constraints, which limit the capability of units to move between scheduled operating levels over short periods of time, can have significant impacts on the solution of the STRS problem. Large, efficient thermal units frequently have the most significant ramp limits in the system. When the difference in system loads in successive periods exceeds unit ramp limits, units which are not significantly ramp-limited, such as smaller (and less efficient) thermal units, combustion turbines, and especially hydro resources, gain additional value because of their ability to match the rapidly changing load. Ramping limits may also constrain the contributions of some units to spinning and operating reserves.

Almost a decade ago, PG&E developed its Hydro-Thermal Optimization (HTO) program. The Lagrangian-relaxation based STRS problem formulation and solution algorithm used in that development have been

described in a previous paper [1]. That formulation included modeling of ramp-constrained thermal units, based on allowing these units to operate at discrete generation levels which could then be represented as states in a dynamic program formulation of the unit scheduling subproblem. This model, while it allowed the ramp-constrained subproblem to be solved as quickly as any other thermal unit subproblem, had the significant disadvantage of not guaranteeing that a ramp-constrained unit would stay committed for its minimum up time. If the STRS problem were to be solved using PG&E's previous ramp-constrained formulation for half-hour or fifteen-minute subintervals, rather than one-hour subintervals as at present, many more thermal units would have to be ramp-limited, and problems with the state space approach would be exacerbated. Another paper's proposed implementation has expanded the dynamic program's state space to include both up time and discretized generation level [2]. But as was pointed out in that paper, the expanded state space causes an order-of-magnitude increase in overall solution time. It should also be noted that the method requires an order-of-magnitude increase in storage requirements for the DP solution.

In this paper, the STRS problem formulation is expanded to explicitly include every individual resource ramp constraint. This approach is essentially that presented in Guan [3], which relaxes and attaches a multiplier to each ramp constraint in the problem formulation. Algorithmically, the dual optimization performed here corresponds closely to that presented in [3]. However, our computational experience was initially somewhat less favorable than was indicated in that paper. We hypothesize that this is due in part to the frequency of ramp violations during startup and shutdown, which we addressed by constraining generation in the ramp-constrained unit subproblems. Therefore, to extend that method we follow another recent paper [4] which treats ramp constraints explicitly by including startup and shutdown ramp constraints, which may constrain unit operations to either a single trajectory of a 'cone' of restricted trajectories. We will describe our implementation of the addition of supplementary trajectory constraints to the ramp-constrained unit subproblems, which improves the efficiency with which a feasible unit schedule is obtained. We will also describe the modified

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unit subproblem solved, and the constraints placed on the economic dispatch in order to maintain feasibility of the unit schedules.

In [4], a system constraint was added to the problem formulation requiring that the sum of ramp limits of committed units be at least sufficient to meet the change in system load from one hour to the next. Such a constraint takes the same form as spinning and operating reserve constraints, and hence the Lagrangian relaxation solution method can yield interesting shadow-price information, in the form of multipliers, about the value of ramp capability in periods when the constraint is binding. We discuss in our conclusions the interpretation of the analogous multipliers for the individual resource ramp constraints.

In the following sections of this paper, we will present a formulation of the STRS problem incorporating unit ramp constraints, describe the solution technique for the modified formulation, give test results for this solution technique, and suggest future applications and extensions of this work.

II Problem formulation

The PG&E power system faces three different kinds of ramp constraints for some steam units: ramp constraints under normal operating condition, startup ramp constraints, and shutdown ramp constraints, which will be defined later.

The STRS problem is formulated here as a thermal unit commitment problem with ramp constraints. The three types of ramp constraints are incorporated into the formulation presented.

In the development the following standard notation will be used. Additional symbols will be introduced when necessary.

i : index for the number of units ($i = 1, \dots, I$);

t : index for time ($t = 0, \dots, T$);

u_{it} : zero-one decision variable indicating whether unit i is up or down in time period t ;

x_{it} : state variable indicating the status of unit i in time period t (length of time unit has been up or down);

t_i^{on} : the minimum number of periods unit i must remain on after it has been turned on;

t_i^{off} : the minimum number of periods unit i must remain off after it has been turned off;

p_{it} : state variable indicating the amount of power unit i is producing in time period t ;

$C_i(p_{it})$: fuel cost for operation unit i at output level p_{it} in time period t ;

$S_i(x_{i,t-1}, u_{it}, u_{i,t-1})$: transition or start-up cost associated with turning on unit i at the beginning of time period t ;

D_t : demand requirement in time period t ;

R_t : spinning capacity requirement in time period t ;

p_i^{\min} : minimum rated capacity of unit i ;

p_i^{\max} : maximum rated capacity of unit i ;

The unit commitment problem is formulated as the following mixed-integer programming problem:

$$(P) \min_{u_{it}, x_{it}, p_{it}} \sum_{t=1}^T \sum_{i=1}^I [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] \quad (1)$$

subject to the demand constraints,

$$\sum_{i=1}^I p_{it}u_{it} = D_t, \quad t = 1, \dots, T, \quad (2)$$

the spinning capacity constraints,

$$\sum_{i=1}^I p_i^{\max}u_{it} \geq R_t, \quad t = 1, \dots, T, \quad (3)$$

and the ramp constraints,

$$\delta_i(u_{i,t-1}, x_{it}) \leq p_{it}u_{it} - p_{i,t-1}u_{i,t-1} \leq \Delta_i(u_{i,t-1}, x_{it}), \quad t = 1, \dots, T; i = 1, \dots, I, \quad (4)$$

where $\delta_i(u_{i,t-1}, x_{it})$ and $\Delta_i(u_{i,t-1}, x_{it})$ are functions of $u_{i,t-1}$ and x_{it} and are to be explained later. There are other unit constraints such as the capacity constraints,

$$p_i^{\min} \leq p_{it} \leq p_i^{\max}, \quad i = 1, \dots, I; t = 1, \dots, T, \quad (5)$$

the minimum up/down time constraints,

$$u_{it} = \begin{cases} 1, & \text{if } 1 \leq x_{it} < t_i^{\text{on}}, \\ 0, & \text{if } -1 \geq x_{it} > -t_i^{\text{off}}, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \quad (6)$$

Consider three kinds of ramp constraints for unit i : startup ramp constraint, shutdown ramp constraint and operating ramp constraint.

- Startup ramp constraint with time τ_i from startup to full availability: When an off-line unit is turned on, it takes τ_i periods with fixed increasing rate in generation p_i^{\min}/τ_i to reach the minimum rated capacity of unit i . For example, let $u_{i,t-1} = 0$ and $u_{it} = 1$, $\delta_i(u_{i,t+j-1}, x_{i,t+j}) = \Delta_i(u_{i,t+j}, x_{i,t+j}) = p_i^{\min}/\tau_i$, $j = 0, \dots, \tau_i - 1$.

- Shutdown ramp constraint with time v_i from full availability to shutdown: When an on-line unit is turned off, its generation level has to be reduced to the minimum rated capacity, p_i^{\min} , subject to the operating ramp constraint defined next, then from p_i^{\min} , it takes v_i periods with fixed decreasing rate in generation p_i^{\min}/v_i to reduce to zero. For example, let $u_{i,t} = u_{i,t+1} = \dots = u_{i,t+v-1} = 1$ and $u_{i,t+v} = 0$ (with $p_{i,t} = p_i^{\min}$), $\delta_i(u_{i,t+j-1}, x_{i,t+j}) = \Delta_i(u_{i,t+j-1}, x_{i,t+j}) = -p_i^{\min}/v_i$, $j = 0, \dots, v_i$.
- Operating ramp constraint Δ_i : the difference of the generation level of unit i on any two successive on-line periods (not in either startup or shutdown periods) is bounded by Δ_i . For example, let $u_{i,t-1} = u_{i,t} = 1$, then $\Delta_i(u_{i,t-1}, x_{i,t}) = -\delta_i(u_{i,t-1}, x_{i,t}) = \Delta_i$.

III Solution procedure: dual optimization phase

The Lagrangian Relaxation approach relaxes not only the demand constraints and the spinning capacity constraints but also the ramp constraints by using Lagrange multipliers. The problem is then decomposed into I subproblems. Let λ_t , μ_t , ξ_{it} and ζ_{it} ($t = 1, \dots, T$, $i = 1, \dots, I$) be the corresponding nonnegative Lagrange multipliers to (2), (3) and (4), we have the following dual problem: (note: underlined variables are vectors.)

$$(D) \quad \max_{\lambda \geq 0, \underline{\mu} \geq 0, \underline{\xi} \geq 0, \underline{\zeta} \geq 0} \theta(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta}), \quad (7)$$

where $\theta(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta}) \equiv$

$$\begin{aligned} & \min_{u_{it}, x_{it}, p_{it}} \sum_{t=1}^T \sum_{i=1}^I [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1})] \\ & + \sum_{t=1}^T [\lambda_t (D_t - \sum_{i=1}^I p_{it}u_{it}) + \mu_t (R_t - \sum_{i=1}^I p_i^{\max}u_{it})] \\ & + \sum_{t=1}^T \sum_{i=1}^I \xi_{it} (p_{it}u_{it} - p_{i,t-1}u_{i,t-1} - \Delta_i(u_{i,t-1}, x_{it})) \\ & + \sum_{t=1}^T \sum_{i=1}^I \zeta_{it} (p_{i,t-1}u_{i,t-1} - p_{it}u_{it} + \delta_i(u_{i,t-1}, x_{it})) \end{aligned} \quad (8)$$

subject to initial conditions, ramp constraints and the unit constraints (5) and (6). A rearrangement of the terms in (8) reveals its separable nature.

$$\theta(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta}) = \sum_{i=1}^I \theta_i(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta}) + \sum_{t=1}^T (\lambda_t D_t + \mu_t R_t), \quad (9)$$

where $\theta_i(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta}) =$

$$\min_{u_{it}, x_{it}, p_{it}} \sum_{t=1}^T [C_i(p_{it})u_{it} + S_i(x_{i,t-1}, u_{it}, u_{i,t-1}) - \lambda_t p_{it}$$

$$\begin{aligned} & - \mu_t p_i^{\max} u_{it} + \sum_{t=1}^T \xi_{it} (p_{it}u_{it} - p_{i,t-1}u_{i,t-1} - \Delta_i(u_{i,t-1}, x_{it})) \\ & + \sum_{t=1}^T \zeta_{it} (p_{i,t-1}u_{i,t-1} - p_{it}u_{it} + \delta_i(u_{i,t-1}, x_{it})) \end{aligned} \quad (10)$$

Once again, the minimization in (10) is subject to initial conditions, ramp constraints and the unit constraints (5) and (6).

The subgradient of θ in $(\lambda_t; \mu_t; \xi_{it}; \zeta_{it})$ is given by

$$\begin{aligned} & (D_t - \sum_{i=1}^I p_{it}; R_t - \sum_{i=1}^I p_i^{\max}u_{it}; p_{it}u_{it} - p_{i,t-1}u_{i,t-1} - \\ & \Delta_i(u_{i,t-1}, x_{it}); p_{i,t-1}u_{i,t-1} - p_{it}u_{it} + \delta_i(u_{i,t-1}, x_{it})) \end{aligned}$$

where (u_{it}, p_{it}, x_{it}) minimizes $\theta(\underline{\lambda}, \underline{\mu}, \underline{\xi}, \underline{\zeta})$ subject to initial conditions, ramp constraints and the unit constraints (5) and (6).

The function $\theta : R^+ \times R^+ \times R^+ \times R^+ \rightarrow R$ is concave and the multiplier updating rule based on the subgradient of θ in $(\lambda_t, \mu_t, \xi_{it}, \zeta_{it})$ at iteration k (denoted by superscripts of variables) has the form

$$\lambda_t^{k+1} = \max[0, \lambda_t^k + s^k (D_t - \sum_{i=1}^I p_{it}^k u_{it}^k)] \quad (11)$$

$$\mu_t^{k+1} = \max[0, \mu_t^k + s^k (R_t - \sum_{i=1}^I p_i^{\max} u_{it}^k)] \quad (12)$$

$$\begin{aligned} \xi_{it}^{k+1} = \max[0, \xi_{it}^k + s^k (p_{it}^k u_{it}^k - p_{i,t-1}^k u_{i,t-1}^k - \\ \delta_i(u_{i,t-1}^k, x_{it}^k))] \end{aligned} \quad (13)$$

$$\begin{aligned} \zeta_{it}^{k+1} = \max[0, \zeta_{it}^k + s^k (p_{i,t-1}^k u_{i,t-1}^k - p_{it}^k u_{it}^k - \\ \Delta_i(u_{i,t-1}^k, x_{it}^k))] \end{aligned} \quad (14)$$

where s^k is a scalar step-size, which for $k \rightarrow \infty$, must satisfy $s^k \rightarrow 0$ and $\sum_{l=1}^k s^l \rightarrow \infty$ if convergence is to be assured [5]. Consider an example schedule of unit i in Table 1 over $J+2$ periods ($t-1, \dots, t+J$, with $J > \tau_i + v_i$). Unit i is off-line initially and is committed on period $t+1$ and decommitted on $t+J$. The subgradients of θ in $\xi_{i,t+j}$ and $\zeta_{i,t+j}$ are calculated as follows.

1. For $j = 1$,

$$\begin{aligned} \xi_{i,t+j} &= p_{i,t+j}u_{i,t+j} - p_{i,t+j-1}u_{i,t+j-1} - \\ & \Delta_i(u_{i,t+j-1}, x_{i,t+j}) \\ &= p_{i,t+j} - \Delta_i(u_{i,t+j-1}, x_{i,t+j}) \\ &= p_i^{\min}/\tau_i - p_i^{\min}/\tau_i \\ &= 0 \end{aligned} \quad (15)$$

Similarly, $\zeta_{i,t+j} = 0$.

2. For $1 \leq j \leq \tau_i$,

$$\begin{aligned}\xi_{i,t+j} &= p_{i,t+j}u_{i,t+j} - p_{i,t+j-1}u_{i,t+j-1} - \Delta_i(u_{i,t+j-1}, x_{i,t+j}) \\ &= p_{i,t+j} - p_{i,t+j-1} - \Delta_i(u_{i,t+j-1}, x_{i,t+j}) \\ &= (j-1)p_i^{\min}/\tau_i - jp_i^{\min}/\tau_i + p_i^{\min}/\tau_i \\ &= 0\end{aligned}\quad (16)$$

Again, $\zeta_{i,t+j} = 0$.

3. The other cases, $j > \tau_i$ and $J \geq j \geq J - v$, are left to the reader.

The results are summarized in Table 1.

Table 1: Subgradients of θ in ξ and ζ of an example unit schedule

	$u_{i,t+j}$	Subgradient of θ in $\xi_{i,t+j}$	Subgradient of θ in $\zeta_{i,t+j}$
$j = -1$	0	0	0
$j = 0$	0	0	0
$j = 1$	1	0	0
$j \leq \tau_i$	1	0	0
$j > \tau_i$	1	$p_{i,t+j} - p_{i,t+j-1} - \Delta_i$	$p_{i,t+j-1} - p_{i,t+j} - \Delta_i$
$J > j \geq J - v_i$	1	0	0
$j = J$	0	0	0

III.1 Solution of ramp constrained subproblems

Each subproblem (10) can be solved using dynamic programming. The transition diagram for subproblem i is in Figure 1. In Figure 1, t_1 , t_2 and $-t_4$ correspond to τ_i , t_i^{up} and $-t_i^{\text{off}}$, respectively. The state corresponding to cold startup is represented by $-t_5$ in Figure 1. So the states between $-t_5$ to t_2 in Figure 1 correspond to the range of state variable x_{it} . In order to accommodate the shutdown ramp constraints, extra states are needed. They are the states between t_2+1 to t_3 (with $t_3-t_2 = v_i$) in Figure 1. Also note that the generation levels at states t_1 and t_2+1 are both p_i^{\min} .

For an on-line period, say $u_{it} = 1$, the continuous variable p_{it} can be determined by solving the following one-dimensional optimization problem.

$$\min_{p_i^{\min} \leq p_{it} \leq p_i^{\max}} C_i(p_{it}) - (\lambda_t - \xi_{it} + \xi_{i,t+1} + \zeta_{it} - \zeta_{i,t+1})p_{it}. \quad (17)$$

Depending on the value of the operating ramp constraint Δ_i relative to a unit's normal operating range $p_i^{\max} - p_i^{\min}$, the unit's operating maximum is modified as a function of x_{it} . In the case of startup, for $x_{it} \geq \tau_i$, and $x_{it} - \tau_i \leq (p_i^{\max} - p_i^{\min})/\Delta_i$, we have

$$p_i^{\max}(x_{it}) = p_i^{\min} + (x_{it} - \tau_i)\Delta_i. \quad (18)$$

The same holds true for periods before entering into the shutdown ramp states; for these periods one may either define additional shutdown states as part of the overall shutdown trajectory as mentioned above, or use multipliers to enforce the ramp constraints as one would during normal operations. With these supplementary startup and shutdown constraints (18), the subproblem solution converges more quickly to unit schedules satisfying ramp constraints, and maintains ramp feasibility throughout the algorithm.

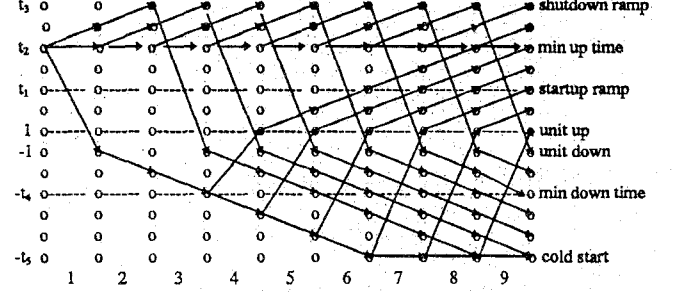


Figure 1: The state transition diagram

IV Finding a feasible solution

Described in general terms, our approach to finding a feasible schedule, based on a near-optimal solution to the Lagrangian dual problem, has three parts. They are illustrated in Figure 2.

(1) A feasible solution is found by projecting the subgradient onto the unsatisfied capacity constraints and updating only the multipliers associated with these constraints. It is hoped, though as far as we know there is no theoretical guarantee, that this approach will yield a feasible schedule and an associated set of multipliers which are close to the schedule and multipliers obtained in the dual optimization.

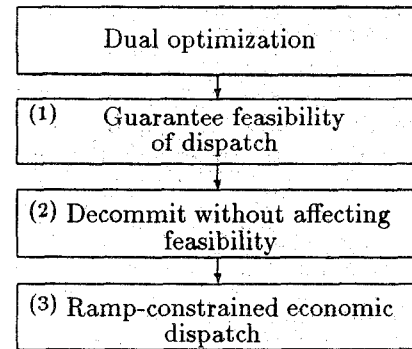


Figure 2: Solution procedure

During this phase we seek to respect the ramp constraints in exactly the same way that we respect them during the dual optimization. Thus, the ramp constraint multipliers are updated according to the same procedure given above during this phase.

We note that this part of the 'feasibility phase' is in fact very similar to the 'maximum capacity constraint' approach described in [3] and [6] to obtain a feasible schedule using Lagrangian relaxation.

(2) Because of the inherently unpredictable response of the system as a whole to the updated set of multipliers, we next check to see whether the feasible schedule can be improved by decommitting units without affecting feasibility.

A number of approaches can be taken to the 'optimal decommitment' problem [7]. The observance of ramp constraints can of course be guaranteed by not allowing decommitment of ramp-constrained units, and not allowing decommitment of any units when decommitment would cause other units to violate ramp constraints. However, it is possible to further refine decommitment so that a less conservative approach can be taken.

(3) An economic dispatch subject to ramp constraints is performed. The economic dispatch is performed for each hour of the schedule in succession, starting with the first hour, so ramp constraints are satisfied based on the principle that a ramp-constrained unit's dispatch must be feasible with respect to both its economic dispatch in the preceding hour, and its apparent generation level in the succeeding hour based on the previously obtained feasible schedule.

V Test results

Figure 3(a)-3(c) illustrate the effects of ramp rate constraints on a large and efficient steam unit A. With no ramp rate constraint, the unit goes to its maximum generating level of 500 mw in the first hour after startup. With a ramp rate of 150 mw/hour (applied also to startup and shutdown ramp) it is committed an hour earlier, and spends less time at its minimum during the off-peak hour and at its maximum during peak hours, but clearly even so it tracks the unconstrained schedule very closely. A ramp rate of 75 mw/hour, however, causes a major commitment change. The unit is now scheduled a day earlier than in the unconstrained case, and again spends less time at minimum and maximum. We see that the interactions between a ramp rate imposed on a unit and the unit commitment are not necessarily confined to that unit alone.

Empirical experience with our new treatment of ramp constraints has indicated that, surprisingly, the presence of ramp constraints and their associated Lagrange multipliers affects solution time per iteration very little. This result may be due to the fact that we are in fact enforcing ramp feasibility during startup and shutdown

by means of the "supplementary constraints" described above. Also, if the system constraint multipliers change very little between iterations, the ramp constraint multipliers will also have to be changed very little to maintain ramp feasibility.

On the other hand, the effects of ramp constraints on overall solution time are not predictable, because these effects are seen primarily in the number of iterations required in the algorithm's feasibility phase. We have seen cases where the presence of moderate ramp constraints actually improved solution time by reducing the number of iterations needed to find a feasible solution. But as ramp limits are made more and more constraining, the algorithm may begin to have difficulties finding any feasible solution, and in fact none may exist.

For realistic applications, however, we have found the new treatment of ramp constraints to yield very good results.

VI Conclusion

We have presented a Lagrangian-relaxation based algorithm for short-term resource scheduling which includes detailed modeling of individual thermal unit ramp constraints. We have implemented this algorithm in PG&E's HTO program, and have found the method to be computationally efficient. The addition of supplementary startup and shutdown constraints, in the form of operating limits that depend on how long a unit has been up or how soon it will begin to shut down, has aided in the efficient solution of the ramp constrained problem.

We are currently investigating at the interpretation of the ramp constraint multipliers, and the relation between them and the value of ramping capacity to the system as a whole. We expect that the lowest unit ramping multiplier value would provide a bound on the value to the system of ramping capability, given fixed relative operating efficiencies. We are also considering implementing a model combining a system ramping capacity constraint, as in [4], with our current detailed model: the combination may lead to improved solutions and improved solution times. We are also looking at how to replace a suboptimal, static economic dispatch with a dynamic dispatch that optimizes given ramp limits over the scheduling horizon. In the longer term, we are interested in the modeling of emergency versus normal ramp limits; i.e., modeling the normal ramp limit as a "soft" constraint.

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VIII Biographies

Alva J. Svoboda received a B.A. in mathematics from U.C. Santa Barbara in 1980, and an M.S. and Ph.D. in Operations Research from U.C. Berkeley in 1984 and 1992. He has worked on contract as an operations research analyst for Pacific Gas and Electric Co. since 1986. His current research interest is the extension of existing utility operations planning models to incorporate new operating constraints.

Chung-Li Tseng received a B.S. in Electrical Engineering from National Taiwan University in 1988 and a M.S. in Electrical and Computer Engineering from U.C. Davis in 1992. He is currently working toward his Ph.D. in Industrial Engineering and Operations Research at U.C. Berkeley.

Chao-an Li graduated from Electric Power System Department of Moscow Energetic Institute, Moscow, USSR. He has broad interests in power system optimization including hydrothermal coordination, economic dispatch, unit commitment, local forecasting, automatic generation control, power flow and power system state estimation problems. He is currently working on projects related to hydro-thermal optimization for PG&E.

Raymond B. Johnson received his B.A. in 1976 in Electrical Sciences from Trinity College, Cambridge University, and a Ph.D. in Electrical Engineering from Imperial College, London University in 1985. His professional

experience includes positions as a power system design engineer with Hawker Siddeley Power Engineering from 1976 to 1980 and an EMS applications developer with Ferranti International Controls from 1987 to 1989. Since 1989, has been with PG&E where he is currently a Systems Engineering Team Leader responsible for resource scheduling and energy trading applications.

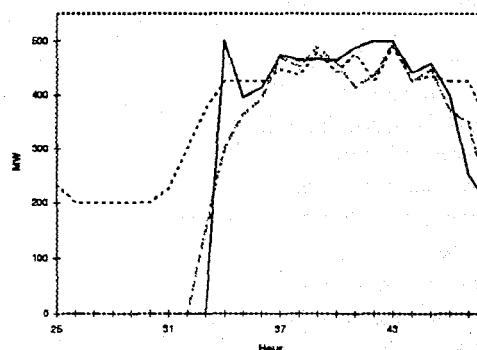


Figure 3(a): Unit A operation on Day 2

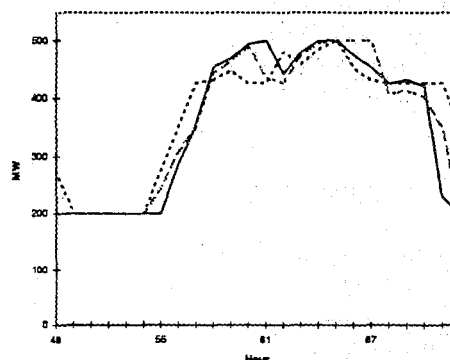


Figure 3(b): Unit A operation on Day 3

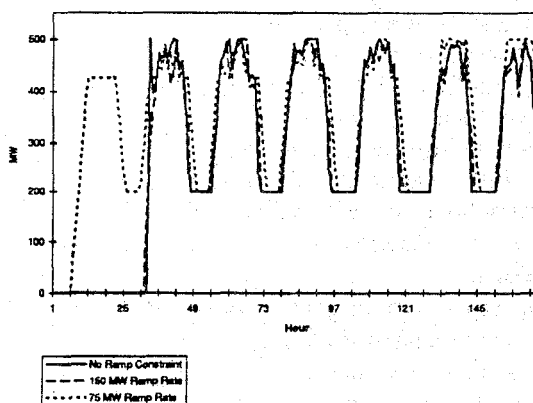


Figure 3(c): Unit A operation

Discussion

M. FOTUHI-FIRUZABAD, S. ABORESHAID AND R. BILLINTON, (Power Systems Research Group, University of Saskatchewan): The authors to be congratulated for presenting an interesting paper on the short-term resource scheduling (STRS) problem. In this paper, a Lagrangian-relaxation based algorithm is proposed which explicitly includes individual thermal unit ramp rate constraints. Other practical constraints such as demand, spinning reserve, minimum up/down time and unit capacity constraints are also considered in the analysis. The authors have done a commendable job in trying to address all these concerns in the application of the proposed method. We would, however, like to seek the authors clarification on the following additional points.

- In Section II, the three different ramp constraints of start up, shut down and normal operating ramp constraints are considered for a given unit i . The operating ramp constraint Δ_i is found using the difference in the generation level of unit i on any two successive on-line time period. Would the authors please indicate whether there is any constraint in the selection of the length of this time period or not. Is there any relationship between the start up or shut down time and the selection of the length of this time period?. How does the length of this time period influence the optimization procedure?
- Equation (1) has two terms, the fuel cost and the start-up cost. It would be appreciated if the authors would clarify how the shut-down cost is included in the optimization procedure.
- In the unit commitment process, it is sometimes necessary to keep some on-line thermal units in the hot state for a few hours rather than shut them down and then start them again depending on the system load variation. This can be done by comparing the shut-down and the start-up costs with the costs associated with keeping the units in a hot state. How are hot stand-by units modeled in the proposed procedure?
- In Section V, the test results show the output profile of a unit for different ramp rate constraints. It would be useful if the authors could provide some information on the effect on the overall system operating cost of variation in the ramp rate constraints.

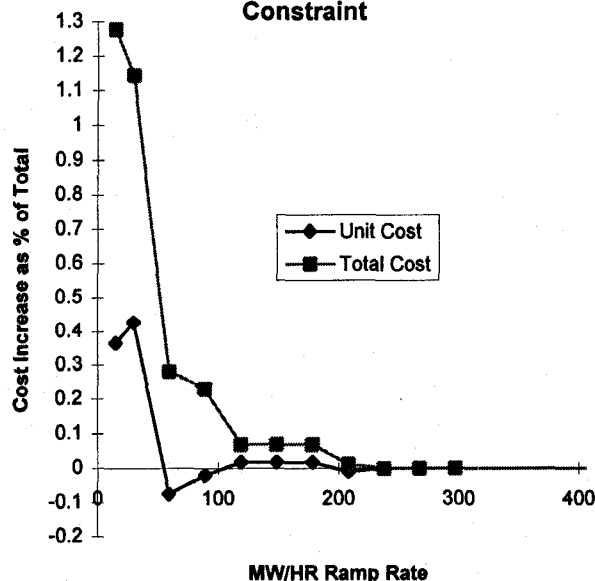
Once again we congratulate the authors for their interesting paper.

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Alva J. Svoboda, Chung-Li Tseng, Chao-an Li and Raymond B. Johnson: The authors thank the discussants for their comments and for raising several points of interest. Our responses to their questions and requests for clarification are as follows:

- 1) The operational short-term scheduling model uses one-hour subperiods over which startup, shutdown and normal operating ramp constraints are defined. We have tested the model using shorter subperiods with durations between 30 and 5 minutes, but have so far lacked access to sufficiently good data regarding these subperiods to put a model with a shorter subperiod into operational use. Thus, startup and shutdown times have not to date directly affected the choice of

Figure A: Cost Increase Over No Ramp Constraint



subperiod. Because a shorter subperiod would imply that many more, perhaps all, units would have to be modeled with ramp constraints, we would expect program execution time to increase.

- 2) Shutdown costs are not at present considered by the model, although shutdown times are.

- 3) Hot standby fuel requirements and stress costs are represented explicitly in HTO's thermal unit modeling, and units can go on hot standby rather than shutting down. However, we have not seen scheduling of hot standby in operational use of the model, either before or after the introduction of our current model of ramp constraints.

- 4) Figure A is illustrative of the effects of variation in one major unit's ramp constraint on overall system costs. In this example, the effects on total costs are relatively insignificant until the ramp constraint is reduced to 100 MW/hour or less, from which point costs rise steeply as ramping flexibility declines. When the unit is not permitted to ramp more than about 10 MW/hour, the problem becomes infeasible because on-peak spinning capacity and off-peak minimum load constraints cannot both be satisfied for the given overall resource configuration. It should be noted that in a single run of the short-term scheduling model, the effects of modifying the ramp constraint are not always predictable because the Lagrangian relaxation model can only guarantee near-optimality to within a small percentage of the true optimum.

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